Simple stochastic model for optical tunneling

A. Ranfagni,[1] R. Ruggeri,[1] P. Sandri,[2] and A. Agresti[3]
[1]*Istituto di Ricerca sulle Onde Eletromagnetiche "Nello Carrara," CNR, Via Panciatichi 64, 50127 Firenze, Italy*
[2]*Scuola di Specializzazione in Ottica dell'Università di Firenze, Firenze, Italy*
[3]*Dipartimento di Fisica dell'Università di Firenze, and Istituto Nazionale di Fisica della Materia, Unitá di Ricerca di Firenze, Firenze, Italy*

(Received 4 September 2001; published 7 March 2002)

A simple model, derived from a Brownian-motion scheme, is capable of interpreting the results of delay-time measurements relative to frustrated total reflection experiments at the microwave scale but also in the visible region. In this framework we also obtain a plausible description of the trajectories (rays) inside the tunneling region, the air gap between two paraffin prisms.

DOI: 10.1103/PhysRevE.65.037601 PACS number(s): 41.20.−q, 02.50.−r

The motion of a particle undergoing quantum tunneling—one of the more nonclassical prediction of quantum mechanics—has long been an open problem in several aspects. Presently, the focus of much tunneling research is to determine the degree to which this quantum concept can be extended to the macroscopic world [1]. In this framework, optical tunneling at the scale of visible [2] or microwave [3] range, demonstrated to be a powerful method of investigation. A model for tunneling based on stochastic processes [4] proved to be capable of interpreting the result of delay-time measurements at the microwave scale [5]. A clear evidence of this nature is here achieved by adopting a Brownian-motion scheme, according to a formulation based on path-integrals methods [6]. In such a way, a good description of experimental results, obtained in frustrated total internal reflection experiments [2,5], can be obtained. Moreover, we can try to determine the shape of the trajectory followed by the system while tunneling: a question lacking of a definite answer [7].

As for the delay or tunneling time, the stochastic model is able to give an original explanation of the experimental results, alternative and even better than the canonical models (mainly the phase-time one adopted in Ref. [2]), including the superluminal behavior, whose interpretation is still a debated and controversial question in both classically forbidden (tunneling) [8], and classically allowed processes [9,10].

According to a semiclassical analysis [11], tunneling occurs when there is not any simultaneous (real) solution of the Euler-Lagrange equation, $\delta S=0$, and Hamilton-Jacobi equation, $E+\partial S/\partial t=0$, S being the action, and E the energy of the particle. The most direct approach to the problem would obtain if one could use complex valued functions $x(t)$, describing the motion of the particle, satisfying the above-mentioned equations in a path integral over complex paths and thereby get a full analog for the path integral of the stationary phase approximation. Despite the initial pessimism, *no one seems to know how to do this* [11], some progress has been obtained following the McLaughlin prescription [12] and recovering many WKB results [13].

Here, as before anticipated, we follow a completely different approach, based on stochastic models which, starting from a pioneering work by Kac [14], then developed by DeWitt-Morette and Foong [15], and subsequently applied to tunneling [16], demonstrated capability of interpreting several physical situations [4,5]. A special stochastic motion is represented by the Brownian motion and, in particular, we have adopted the results of an analysis as given by Feynman and Hibbs at the end of their book [6]. As known, the connection between path integration and Brownian motion is so close that they are nearly indistinguishable [17]. Another important contribution in this context has been given by Dykman *et al.* [18] who investigated the distribution of paths for large fluctuations away from a stable state in the case of an overdamped Brownian particle driven by a periodic force and white noise. This approach, however, does not seem suitable for our cases which refer to *open* systems, that is to tunneling processes undertaken by particles, or better wave pulses, coming from, and going to infinite.

What we are searching for is the shape of the trajectory in tunneling regions, which, as we said before, is unknown. However, in frustrated total reflection experiments [2,5], the shift D of a beam, traversing an air gap between two prisms of refractive index n, was measured by varying the gap width T (see Fig. 1). If we assume that $D(T)$ could be identified with the average path $x(t)$ in Fig. 1, we have solved the

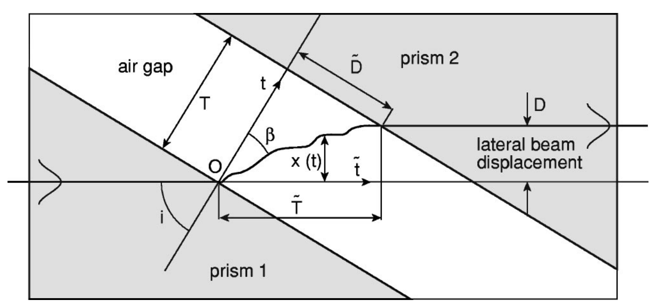

FIG. 1. Schematic representation of the barrier, consisting of an air gap between two paraffin prisms with refractive index $n=1.49$. The incoming microwave beam in the first prism, impinges on the gap with an incidence angle $i=60°$, sufficiently greater than the critical value $i_0=42°$, so that the beam is mainly reflected, while a portion travels the gap and attenuated enters the second prism. The transmitted beam exhibits a lateral displacement D which was measured by varying the gap width T. The quantity $\tilde{D}$, deduced from the displacement D, is a measure of the traversal time. The coordinates t and $\tilde{t}$ have their origin in O.

 ©2002 The American Physical Society

problem of the tunneling trajectories. The plausibility of this assumption will be tested in comparison with the experimental data.

For a Brownian motion—into which we are assimilating the tunneling process—the function $x(t)$ is given by Eq. (12.74) in Ref. [6] and reads as

$$x(t) = (3D - \theta T)(t/T)^2 - (2D - \theta T)(t/T)^3, \tag{1}$$

where D and θ are the displacement and the angular deviation, respectively, of a particle incident perpendicularly on a slab of matter of thickness T, when it emerges from the slab. While traveling through a thickness t it is displaced away from its original trajectory by a distance x (see Fig. 12-1 in Ref. [6]). The assumption is made that θ is always small and that the motion is the result of a large number of collisions each of which has a small effect. In addition, the maximum of the probability distribution, for particles emerging at a given angle θ is found for $D = \theta T/2$. Of course, the average value of Brownian-motion trajectories in a system that presents initially one symmetry (here symmetry with respect to the direction normal to the slab) should keep that symmetry. Different is the case of the tunneling processes we are considering, where the incidence of the beam is oblique with respect to the barrier and the mechanism of the tunneling will tend to privilege trajectories whose deviation with respect to the perpendicular of the barrier have definite sign. According to the experimental evidence, the emergence of the beam in the second prism occurs between the normal to the slab and the prolongation of the incident beam from the first prism, see Fig. 1.

So, in order to adapt Eq. (1) to our geometry we have to substitute θ with $i - \beta$ (i is the incidence angle, β is the mean deviation angle with respect to the perpendicular of the gap), T with $\tilde{T} = T\cos(i-\beta)/\cos\beta$ and t with $\tilde{t} = t\cos(i-\beta)/\cos\beta$, so that $\tilde{t}/\tilde{T}$ remains equal to t/T. Following Refs. [2] and [5], we have that the quantity $\tilde{D}$ in Fig. 1 is a measure of the traversal time τ according to the relation $\tau = n\tilde{D}/c\sin i$, $c\sin i/n$ being the velocity component along the gap of the incident beam. In turn, $\tilde{D}$ is related to D by the trigonometric relation $\tilde{D} = (T\sin i - D)/\cos i$. By substituting into the modified Eq. (1), we have ultimately that the traversal time can be expressed as a function of t, the coordinate perpendicular to the gap (Fig. 1), while β and D are to be considered as moderately adjustable parameters.

A result is shown in Fig. 2 by a curve which fits the experimental data, represented by small squares with their fiducial bars, obtained according to the experimental procedure of Ref. [5]. The curve well reproduces the characteristic behavior with a peak which confirms the prediction of the stochastic model as in Ref. [5]. In the same figure we also show the curve resulting from the phase-time model which results clearly inadequate to fit the experimental data when the phase variation is taken along the direction normal to the slab, even for an incidence angle near to the critical value ($\sim 42°$) [19]. The straight line $v = c$ evidences the superluminal behavior for gap width greater than one centimeter.

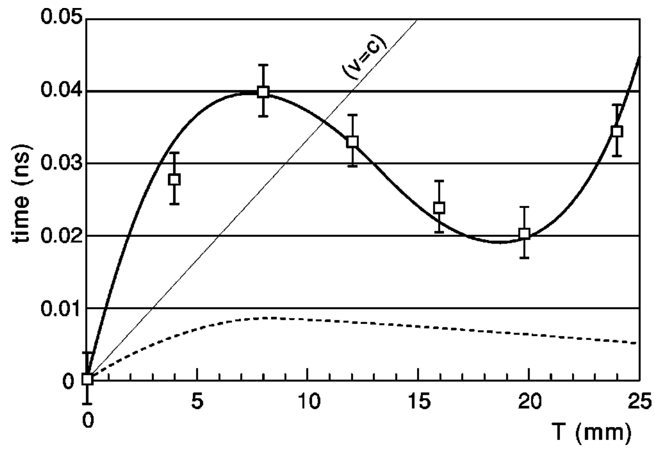

FIG. 2. Traversal time results, as a function of the gap width T between the two prisms, in the case of a frustrated total reflection experiment at the microwave scale with a beam at 9.33 GHz. The continuous curve is deduced from the function $x(t)$, Eq. (1), properly adapted to the geometry of Fig. 1, with parameter values as $D = 22.6$ mm and $\beta = 40.5°$, the dashed curve represents the theoretical prediction according to the phase-time model.

More important, and this is one aspect, is the shape of the trajectory in the gap between the two prisms, as given by the function $x(t)$, when properly collocated in the space of the gap, as in Fig. 3. Here the experimental points refer to three series of measurements: one corresponds to the data shown in Fig. 2 (small squares), another to the data reported in Ref. [5] (small crosses), and a third series of measurements (small triangles). The continuous curve of $x(\tilde{t})$ fits the results rela-

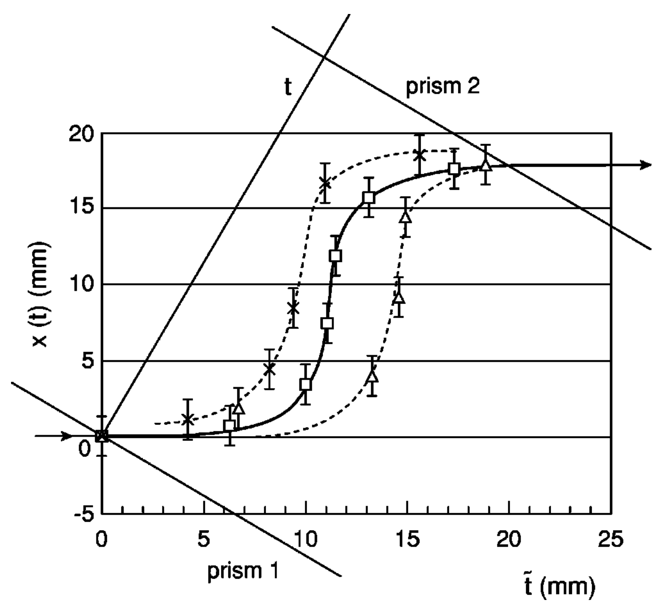

FIG. 3. Plausible shape of the trajectories inside the air gap as deduced from three series of measurements. The continuous curve, which fits the data corresponding to the results of Fig. 2, is the function $x(\tilde{t})$ when situated in the gap space. The dashed curves, which fit the other two series of data, confirm the shape in spite of non-negligible displacements.

tive to Fig. 2, while the dashed lines fit the other data. In every case we obtain a rather plausible and stable behavior which can be assumed as the shape of the trajectories inside the gap. The spreading of the results is acceptable given the delicateness of the experimental procedure. These results refer to microwave measurements, but a similar behavior can be obtained also in the case of the optical experiment of Ref. [2], obtaining a description of their results (not shown here) even better than that reported in Ref. [2], although in that case the discrepancy between the predictions of the two models seems to be less evident.

We can therefore conclude that, in spite of the simplicity of the present approach, we have obtained an acceptable description of the trajectories, as well as of the delay time, in the forbidden region—the air gap between the two paraffin prisms—of an optical experiment at the microwave scale. The plausibility of the model, although rather simplified, resides on its capability to reproduce data of delay time analogously and better than the models previously adopted.

[1] A. Yazdani, Nature (London) **409**, 471 (2001).

[2] B. Ph. and L. Dutriaux, Phys. Rev. Lett. **78**, 851 (1997).

[3] D. Mugnai, A. Ranfagni, and L. Ronchi, Phys. Lett. A **247**, 281 (1998).

[4] A. Ranfagni, R. Ruggeri, and A. Agresti, Found. Phys. **28**, 515 (1998).

[5] A. Ranfagni *et al.*, Phys. Rev. E **63**, 025102 (2001).

[6] R. Feynman and A. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965), Chap. 12.

[7] According to a pure orthodox quantum-mechanical point of view the question itself is perhaps devoid of meaning. In the optical framework, however, some notion can be deduced: see, for instance, Refs. [2] and [3].

[8] M. Mojahedi, E. Schamiloglu, F. Hegeler, and K.J. Malloy, Phys. Rev. E **62**, 5758 (2000).

[9] D. Mugnai, A. Ranfagni, and R. Ruggeri, Phys. Rev. Lett. **84**, 4830 (2000). See also P. Ball, *Faster than Light* (Nature Science update, May 30, 2000) and *Going over c* (Nature Science update, July 20, 2000).

[10] L.J. Wang, A. Kuzmich, and A. Dogariu, Nature (London) **406**, 277 (2000).

[11] L. S. Schulman, *Techniques and Applications of Path Integration* (Wiley, New York, 1981), Chap. 18.

[12] D. McLaughlin, J. Math. Phys. **13**, 1099 (1972).

[13] A. Ranfagni and D. Mugnai, Nuovo Cimento D **11**, 53 (1989).

[14] M. Kac, Rocky Mt. J. Math. **4**, 497 (1974), reprinted from Magnolia Petroleum Company Colloquium Lectures in the Pure and Applied Sciences, N. 2, October, 1956.

[15] C. DeWitt-Morette and S.K. Foong, Phys. Rev. Lett. **62**, 2201 (1989).

[16] D. Mugnai, A. Ranfagni, R. Ruggeri, and A. Agresti, Phys. Rev. Lett. **68**, 259 (1992).

[17] Ref. [11], Chap. 19.

[18] M.I. Dykman, D.G. Luchinsky, P.V.E. McClintock, and V.N. Smelyankiy, Phys. Rev. Lett. **77**, 5229 (1996).

[19] In this condition the phase-time delay turns out to have its maximum value and decreases by increasing the incidence angle. For an incidence angle of 60° the phase-time delay is near to zero. See D. Mugnai, A. Ranfagni, and L. Ronchi, *Tunneling Time in the Case of Frustrated Total Reflection*, Atti Fond. Giorgio Ronchi **53**, 777 (1998).